LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – PHYSICS

SECOND SEMESTER – APRIL 2010

PH 2811 / 2808 - QUANTUM MECHANICS

Date & Time: 19/04/2010 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

Part – A (10 x 2 = 20 Marks)

(Answer all questions)

1. If [X , Y] = 1, find [X , Y^2].
2. Show that commuting operators have simultaneous eigen functions.
3. Express position and momentum operators of a linear harmonic oscillator in terms of the operators 'a' and '$a^†$'
4. What are the eigen values of the parity operator? Show that the parity operator can have only two eigen values.
5. Express angular momentum operator $\hat{L}^2$ in terms of $\hat{r}$ and $\hat{p}$, where $\hat{r}$ is position operator and $\hat{p}$ is the momentum operator.
6. If $|\alpha_1\rangle = \begin{bmatrix} cos\theta \\ -sin\theta \end{bmatrix}$ and $|\alpha_2\rangle = \begin{bmatrix} sin\theta \\ cos\theta \end{bmatrix}$ show that $\langle\alpha_i|\alpha_j\rangle = \delta_{ij}$ and $\sum_i |\alpha_i\rangle\langle\alpha_i| = 1$.
7. Show that σ^2 =3, where σ is the spin operator.
8. Show that $e^{i(\sigma.n)\theta/2} = \cos\left(\frac{\theta}{2}\right) + i\,(\sigma.n)\sin\left(\frac{\theta}{2}\right)$
9. What is the first order correction to energy in the case of time – independent perturbation for a non-degenerate energy level?
10. What is the usefulness of variation method? On what assumption is it based?

Part – B (4 x 7.5 = 30 Marks)

(Answer any four questions)

11. Obtain Heisenberg's uncertainty relation, using commutation bracket algebra.
12. a. If p and q are momentum and position operators and a = [λq + i($\frac{p}{\lambda}$ + μq)]/$\sqrt{2\hbar}$, where λ and μ are real, estimate [a , $a^†$]. (5)

    b. If $a|n\rangle = \sqrt{n}\,|n-1\rangle$ and $a^† a|n\rangle = n\,|n\rangle$, find the action of $a^†$ on |n> (2.5)
13. a.The base vectors of a representation are $\begin{bmatrix}1\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix}$. Construct the transformation matrix 'U' for transforming these vectors in to another representation having base vectors $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$ and $\frac{1}{\sqrt{2}}\begin{bmatrix}-1\\1\end{bmatrix}$ (5)

    b.If an operator A = $\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$ in the first representation, what is its form in the second one? (2.5)
14. Show that $e^{i\sigma_x\pi/2}\sigma_y e^{-i\sigma_x\pi/2}$ = - σ_y, where σ_x & σ_y are Pauli's spin matrices (4)

    Show that for Pauli's spin matrices, $\sigma_i\sigma_j + \sigma_j\sigma_i = 2\delta_{ij}$ (3.5)
15. Obtain the first order perturbation equation and the corrected wave functions of the system.

Part – C (4 x 12.5 = 50 Marks)

(Answer any four questions)

16. Obtain Newton's second law of motion from Ehrenfest's theorem.
17. Find the transmission coefficient of a particle moving along the x-axis encountering a potential barrier of breadth 'a' and height V_0, if the energy of the particle E < V_0
18. Define time reversal operator $\hat{T}$. How does it affect the Hamiltonian of a system? What is its effect on the commutator $[x,\ p_x\ ]$ and how does the Schrodinger wave equation transform under time reversal? Find the norm of $\hat{T}$
19. Obtain the matrix representation of operators J^2, J_x, J_y, J_z, J_+ & J_- for a particle with j =1

20. Obtain the ground state energy of the Hydrogen molecule using the variation method.

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